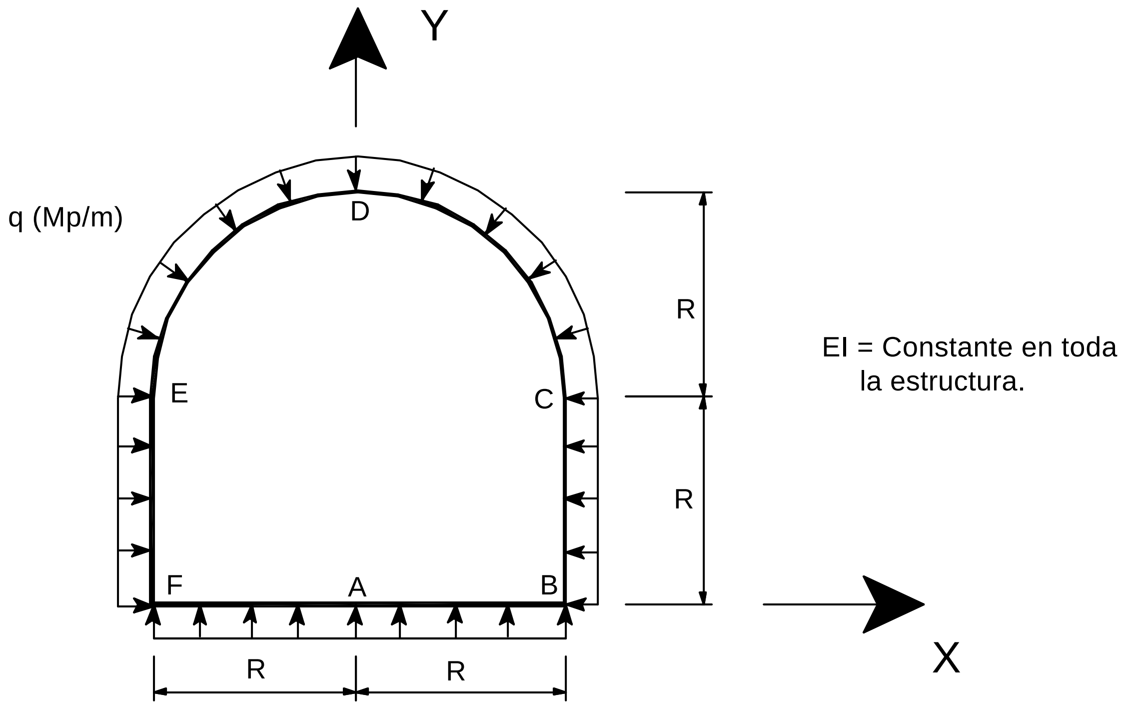
EJERCICIO Nº 3 - G7

En la estructura que se muestra, se pide :

a) Valores del **momento flector** y **axil** en el punto **A**.

b) Diagrama de momento flector de la estructura, indicando sus valores en los puntos más significativos.



SOLUCION

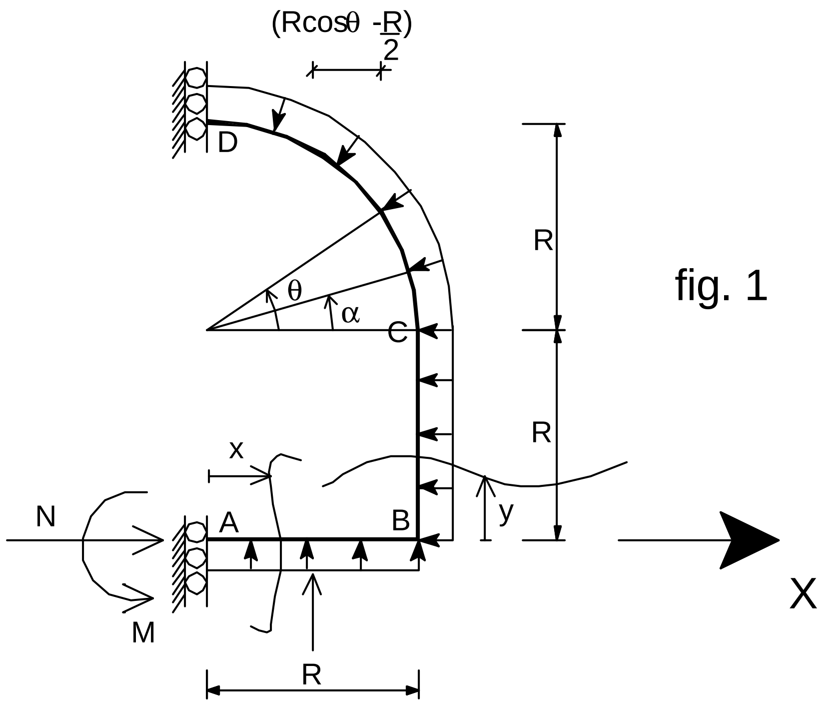


fig. 1

TRAMO	INTERVALO	MOMENTO FLECTOR (M*)
AB	$0 \le x \le R$	$M^* = M - \frac{qx^2}{2}$ { Para x=R => $M^* = M - \frac{qR^2}{2}$
BC	$0 \le y \le R$	$M^* = M + N * y - \frac{qR^2}{2} - \frac{qy^2}{2}$ Para y = 0 => $M^* = M - \frac{qR^2}{2}$ Para y = R => $M^* = M + NR - qR^2$
CD	$\alpha \le \theta \le \frac{\pi}{2}$	$M^* = M + NR(1 + sen\theta) - qR^2\left(\cos\theta - \frac{1}{2}\right) +$ $- qR^2\left(\frac{1}{2} + sen\theta\right) - \int_0^\theta qR^2 d\alpha[sen(\theta - \alpha)]$ M* = M + (1+senθ)(NR-qR²) Para θ = 0, M* = M + NR – qR²

$$(-)qR^2\int_0^{\theta} sen(\theta-\alpha)^{(-)}\,d\alpha = -qR^2\left[-\cos(\theta-\alpha)\right]_0^{\theta} = qR^2\left[1-\cos\theta\right]$$

$M^*_{(\text{tramo CD})}$ = M + NR(1+senθ)-qR^2(senθ + cosθ)-qR^2[1-cosθ]

$M^*_{(\text{tramo CD})}$ = M + NR(1+senθ) –qR^2(1+senθ)

Aplicando BRESSE:

$$\theta_D = 0 = \int \frac{M}{EI}ds \quad => \quad \boxed{\int M^* ds = 0} \quad (1)$$

$$m_D = 0 = -\int \frac{M^*}{EI}(2R-y)dS = 0$$

$$2R\overset{=0}{\cancel{\int MdS}} - \int M^* ydS = 0 \quad => \quad \boxed{\int M^* ydS = 0} \quad (2)$$

Aplicando (1) en fig.1, se tiene:

$$0 = \underbrace{\int_0^R\left(M-\frac{qx^2}{2}\right)dx}_{I_1} + \underbrace{\int_0^R\left(M+N\times y-\frac{qR^2}{2}-\frac{qy^2}{2}\right)dy}_{I_2} + \underbrace{\int_0^{\pi/2}\left[M+(1+sen\theta)(NR-qR^2)\right]Rd\theta}_{I_3}$$

$$I_1 = MR - \frac{qR^3}{6}$$

$$I_2 = MR + \frac{NR^2}{2} - \frac{qR^3}{2} - \frac{qR^3}{6} = MR + \frac{NR^2}{2} - \frac{2}{3}qR^3$$

$$I_3 = \frac{\pi}{2}RM + (NR^2 - qR^3)\left[\theta-\cos\theta\right]_0^{\pi/2}$$

Reemplazando:

$$R\left(2+\frac{\pi}{2}\right)M + R^2\left(\frac{3}{2}+\frac{\pi}{2}\right)N + R^3\left(-\frac{11+3\pi}{6}\right)q = 0$$

$$\boxed{3{,}57M + 3{,}07RN = 3{,}4qR^2} \quad (3)$$

Aplicando (2) en la fig 1, se tiene:

$$\int_0^R\left(M-\frac{qx^2}{2}\right)\overset{=0}{y}\,dx+\underbrace{\int_0^R\left(M+Ny-\frac{qR^2}{2}-\frac{qy^2}{2}\right)y\,dy}_{I_4}+\underbrace{R\int_0^{\pi/2}\left[M+(1+sen\theta)(NR-qR^2)\right]y\,d\theta}_{I_5}$$

$$I_4=\frac{MR^2}{2}+\frac{NR^3}{3}-\frac{qR^4}{4}-\frac{qR^4}{8}$$

$$I_5=R^2\int_0^{\pi/2}\left[M+Nsen\theta+(NR-qR^2)(1+sen\theta)^2\right]d\theta$$

$$I_5=R^2\left[\frac{\pi}{2}M+M+(NR-qR^2)\times 4{,}36\right]$$

Reemplazando:

$$R^2\left(\frac{1}{2}+\frac{\pi}{2}+1\right)M+R^3\left(\frac{1}{3}+4{,}36\right)N=\left(\frac{3}{8}+4{,}36\right)qR^4$$

$$\boxed{3{,}07M + 4{,}69RN = 4{,}73qR^2} \qquad (4)$$

De (3) y (4)

$$M=\frac{\begin{vmatrix}3{,}4qR^2 & 3{,}07R\\ 4{,}73qR^2 & 4{,}69R\end{vmatrix}}{\begin{vmatrix}3{,}57 & 3{,}07R\\ 3{,}07 & 4{,}69R\end{vmatrix}}=\frac{1{,}42qR^3}{7{,}32R}=0{,}19qR^2$$

$$N=\frac{\begin{vmatrix}3{,}57 & 3{,}4qR^2\\ 3{,}07 & 4{,}73qR^2\end{vmatrix}}{7{,}32R}=\frac{6{,}45qR^2}{7{,}32R}=0{,}88qR$$

$$M_B = M - \frac{qx^2}{2} = 0{,}19qR^2 - q\frac{R^2}{2} = -0{,}31qR^2$$

$M_C = M + NR - qR^2 = 0{,}19qR^2 + 0{,}88qR^2 - qR^2 = 0{,}07qR^2$

$M_D = M + 2(0{,}88qR^2 - qR^2) = -0{,}05qR^2$